THE STABILITY OF TWO CONNECTED PENDANT DROPS

Lev A. Slobozhanin and J. Iwan D. Alexander

Department of Mechanical and Aerospace Engineering and National Center for Microgravity Research on Fluids and Combustion, Case Western Reserve University, Cleveland, Ohio 44106, USA

Summary The stability of an equilibrium system of two drops suspended from circular holes is examined. The drop surfaces are disconnected surfaces of a connected liquid body. For holes of equal radii and identical pendant drops axisymmetric perturbations are always the most dangerous. The stability region for two identical drops differs considerably from that for a single drop. Loss of stability leads to a transition from a critical system of identical drops to a stable system of axisymmetric non-identical. This system of non-identical drops reaches its own stability limit (to isochoric or non-isochoric perturbations). For non-identical drops, loss of stability results in dripping or streaming from the holes. Critical volumes for non-identical drops have been calculated as functions of the Bond number, B. For unequal hole radii, stability regions have been constructed for a set of hole radius ratios, K. The dependence of critical volumes on K and B is analyzed.

EXTENDED SUMMARY

We consider an equilibrium system of two drops suspended from edges of horizontal circular holes of radii r_1 and r_2. The liquid region is connected and the capillary free surface consists only of two axisymmetric drop surfaces, Γ_1 and Γ_2. The solution of this problem is the first important step for the analysis of the stability of the system with a multiple holes of radii $r_1, \ldots r_m$ each associated with pendant drop surfaces $\Gamma_1, \ldots, \Gamma_m$ $(m > 2)$ that are disconnected. Such a system has many technical applications.

If an isolated liquid volume is bounded by a disconnected free surface, the condition that the total liquid volume is conserved must be satisfied:

$$\sum_{i=1}^{m} \int_{\Gamma_i} N_i \, d\Gamma = 0. \tag{1}$$

(N_i is the normal component of perturbations on Γ_i). However, the above constant volume constraint (1) does not require that each connectivity component satisfy a separate volume conservation condition of the form

$$\int_{\Gamma_i} N_i \, d\Gamma = 0 \qquad (i = 1, \ldots, m). \tag{2}$$

(so-called isochoric perturbations).

Consequently, the class of admissible perturbations on each surface component Γ_i $(i = 1, \ldots, m)$ of disconnected surface is larger than for the case when Γ_i is the sole free surface component. That means that level of collective stability for a set of pendant drops may be lower in comparison to individual stability of any of the m single drops the system consists from. The used mathematical approach is based on analysis of the second variation of the potential energy under arbitrary perturbations that satisfy the condition (1).

For holes of equal radii $(r_1 = r_2 = r_0)$ and *identical* pendant drops it is found that axisymmetric perturbations are always the most dangerous, and that the stability region for two identical drops and that for a single drop differ considerably in extent. Dependence of the critical dimensionless volume of the protruded part of each identical pendant drop, V^*, on the dimensionless circular hole radius, R_0 (the capillary constant is used for a normalization) has been constructed along with the dependence of critical dimensionless drop height, H^*, on R_0. It is shown that loss of stability of identical drops does not lead to liquid dripping or streaming out from the holes. Rather, as the total protruded liquid volume increases beyond the stability limit for the system of identical drops, there is a transition from a critical system of identical drops to a stable system of axisymmetric non-identical drops. Upon further increase in the total protruded volume, there will be a continuous evolution of a stable system of non-identical drops. As the volume of one drop exceeds the critical volume per drop (for identical drops), the volume of the other drops decreases below that critical volume. Finally, the system of axisymmetric non-identical drops reaches its own stability limit (to isochoric or non-isochoric perturbations). For non-identical pendant drops, loss of stability results in the liquid dripping or streaming out from the holes. Critical volumes V_1^* and V_2^* for non-identical drops have been calculated as functions of R_0.

The more complicated case of unequal radii r_1 and r_2 has been also examined. The stability regions have been constructed for a set the parameter $K \equiv r_1/r_2$ values $(0 < K < 1)$. Critical volumes V_1^* and V_2^* have been calculated, and their dependence on K and R_2 has been analyzed.

For square packing, two branches exist for any $90° < \theta \leq 180°$. Here $V_p^* < V_r^*$ if $90° < \theta \leq 109°$, and $V_p^* > V_r^*$ if $110° \leq \theta \leq 180°$. This causes different ways of infiltration and drainage in a wide interval of θ. Under infiltration, a layer of a square packed arrangement becomes completely coated by liquid as a result of loss of meniscus stability if $90° < \theta \leq 109°$, and as a result of a pendular gas ring formation if $110° \leq \theta \leq 180°$. Similar conclusions are valid for detachment of liquid from the spheres during drainage: loss of the capillary surface stability for the case of square packing and $71° \leq \theta < 90°$, and formation of a pendular liquid ring in the case of $0 \leq \theta \leq 70°$.

In contrast to analysis of hexagonal packing [2], for square packing the stability boundaries for an equilibrium capillary liquid in contact with spherical array have been determined here with a reasonable accuracy and will be presented as tabulated data. It is notable that according to numerical results the modulus of mean curvature for a free surface with self-tangency is practically independent of θ.

For both hexagonal and square packing there is a hysteresis behavior in quasi-static infiltration and drainage processes. The hysteresis is due to irreversible transitions between configurations that belong to different branches, and between a horizontal free surface and a capillary surface in contact with spheres.

All descriptions are valid for mono-layer packing. It is significant that for any contact angle the menisci corresponding to maximum and minimum capillary pressures never cross spheres from the next layers of three-dimensional packing. We have verified this for both square and hexagonal packing. This suggests that for multilayer packing the extreme capillary pressures are the same as for mono-layer packing. Thus, our results allow for an estimation of the lower limit of penetration pressure for mixed types of packing.

References

[1] G. Mason, N. R. Morrow, J. Colloid Interface Sci. 168 (1994) 130-141.

[2] J. L. Hilden, K. P. Trumble, J. Colloid Interface Sci. 267 (2003) 463-474.